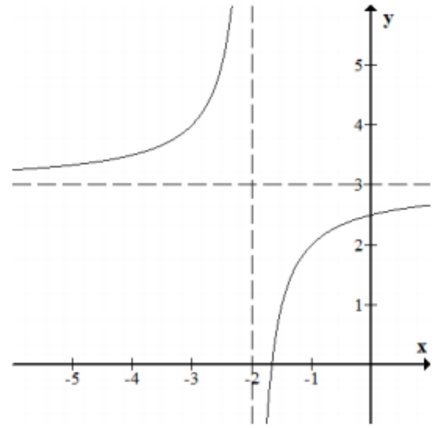
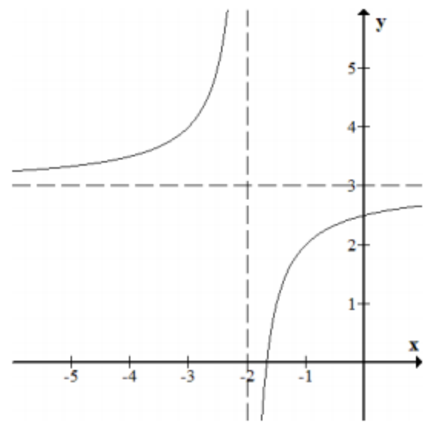
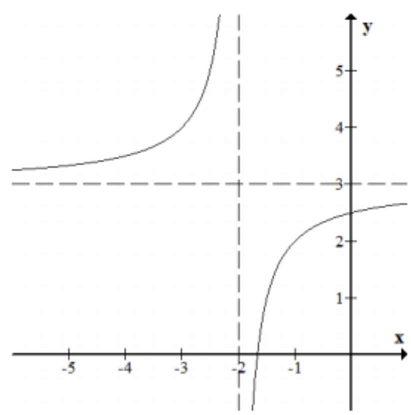
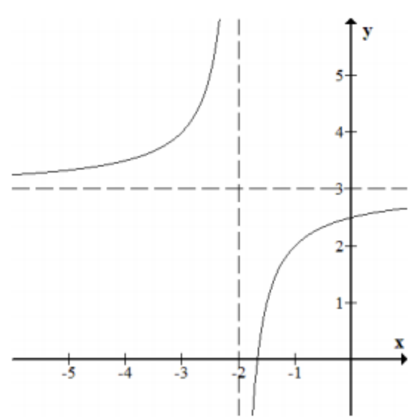
Graphing Rational Functions

Term	How to find it
Vertical Asymptotes – vertical lines which correspond to the zeros of the denominator of the rational function. Tells you how the function will behave as y approaches $\pm\infty$. It will not cross the vertical asymptote as y approaches $\pm\infty$. This function has a vertical asymptote at: $x =$	
Horizontal Asymptote – a horizontal line that tells you a how a function will behave as x approaches $\pm\infty$. It will not cross the horizontal asymptote as x approaches $\pm\infty$. This function has a horizontal asymptote at $y =$	
x - intercept/s – The place where a function crosses the x-axis. Also known as the **solution/s** (x=) of any function. The point where y = 0. Can be found by setting the numerator equal to zero and solving for x, or graphing the rational function in y^1= and graphing y^2=0. Hit graph. Hit 2[nd] →Trace→Intersect, go over the x-intercept and hit enter 3 times. This function an **x-intercept/solution** at:	
y - intercept – The place where the function crosses the y-axis. The point where x = 0. The y – intercept is **never** the solution to any function. The x-value of the x – intercept (see above) is always the solution to any function. Can be found by evaluating the function at f(x) where x = 0. This function has a **y-intercept** at	

How to graph rational functions steps:

1. Find the **vertical asymptotes** by setting the denominator equal to 0 and solving for x. Draw a vertical dotted line at the vertical asymptote and label it as x = whatever it is.

2. Find the **horizontal asymptote** by using the following chart. Draw a horizontal dotted line at the horizontal asymptote and label it as y = whatever it is.

Situation	Example	Horizontal asymptote
top degree < bottom degree	$y = \frac{x+1}{x^2-1}$	Horizontal asymptote is always **$y = 0$**
top degree = bottom degree	$y = \frac{x^2+1}{3x^2+1}$	Find horizontal asymptote by putting the first term of the numerator over the first term of the denominator and simplifying. $y = \frac{x^2}{3x^2} = \frac{1}{3}$ Horizontal asymptote is $\boldsymbol{y = \frac{1}{3}}$
top degree > bottom degree	$y = \frac{x^2+1}{x-3}$	There is no horizontal asymptote. Just put **"none"**.

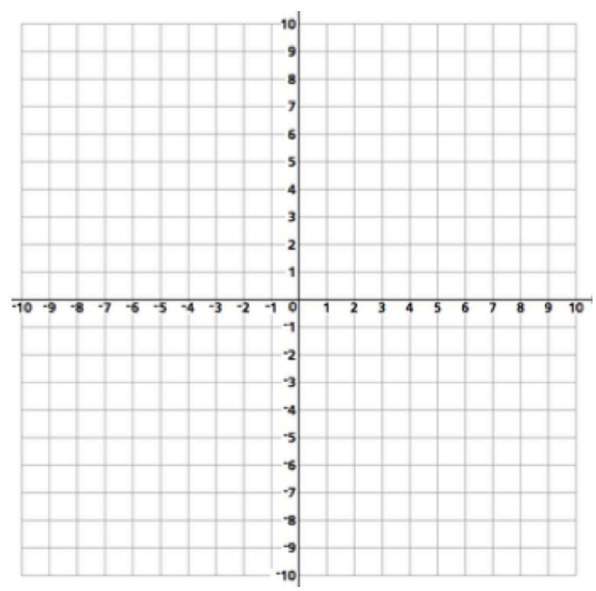
3. Find **x – intercept** by setting the numerator equal to 0 and solving for x, or by using the calculator. Graph the x – intercept.

4. Find the **y-intercept** by evaluating the function, f(x), where x = 0. Graph the y – intercept.

5. **Graph the function** by evaluation the function at different values of x on either side of the vertical asymptote. You can either evaluate the function at different values of x by hand, or graph the function in y^1= in your calculator and hitting 2^{nd} → Graph/Table and using different coordinate values on the table to plot points on your paper.

Example: $y = \frac{x^2+2x-3}{x^2-5x-6}$

x	f(x)



Vertical asymptotes:

Horizontal asymptote:

x – intercept: y – intercept:

Graphing Rational Functions Classwork

Identify the vertical asymptotes, horizontal asymptote, x – intercepts, y – intercept of each.

1) $f(x) = \dfrac{1}{3x^2 + 3x - 18}$

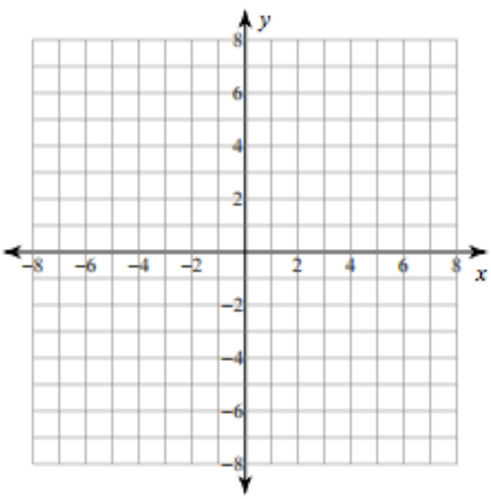
2) $f(x) = \dfrac{x - 2}{x - 4}$

3) $f(x) = \dfrac{x^3 - x^2 - 6x}{-3x^2 - 3x + 18}$

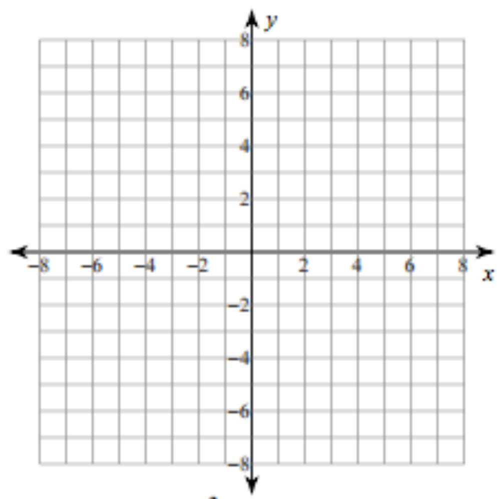
4) $f(x) = \dfrac{x^2 + x - 6}{-4x^2 - 16x - 12}$

Identify the vertical asymptotes, horizontal asymptote, x – intercepts, y – intercept of each.
Then sketch the graph.

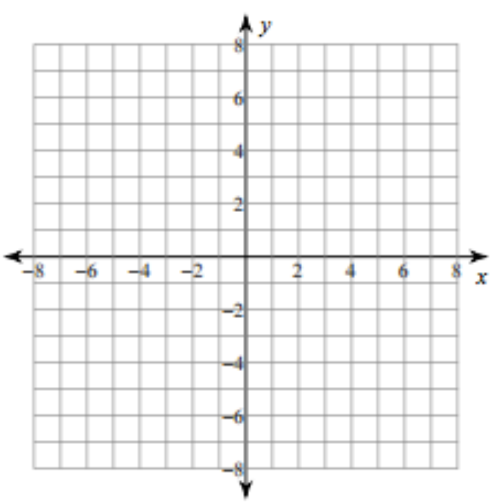
5) $f(x) = -\dfrac{4}{x^2 - 3x}$



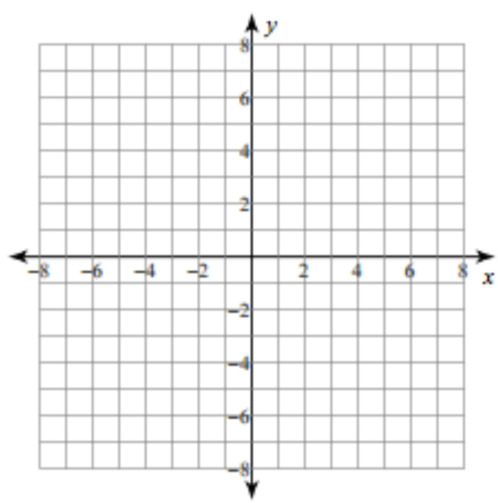
6) $f(x) = \dfrac{x - 4}{-4x - 16}$



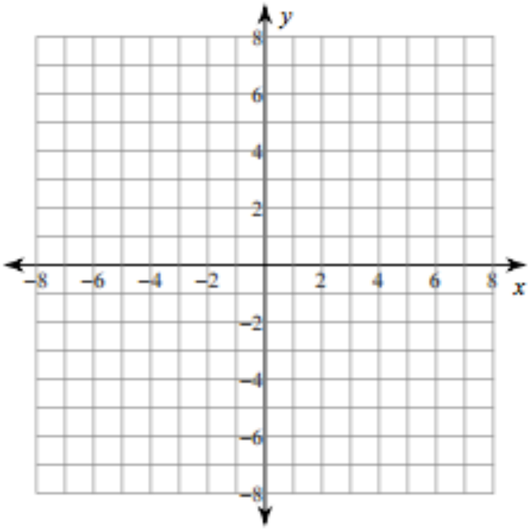
7) $f(x) = \dfrac{x + 4}{-2x - 6}$



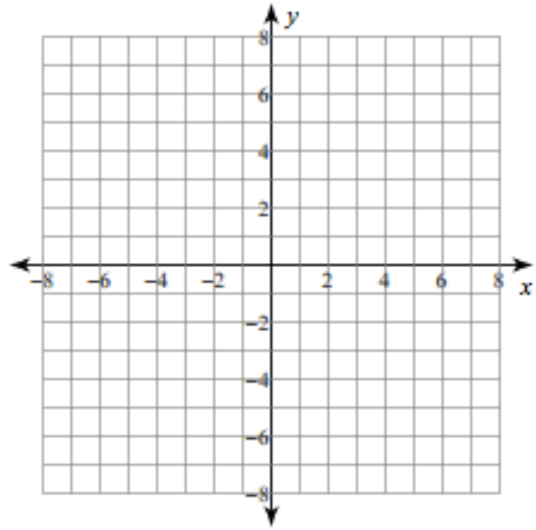
8) $f(x) = \dfrac{x^3 - 9x}{3x^2 - 6x - 9}$



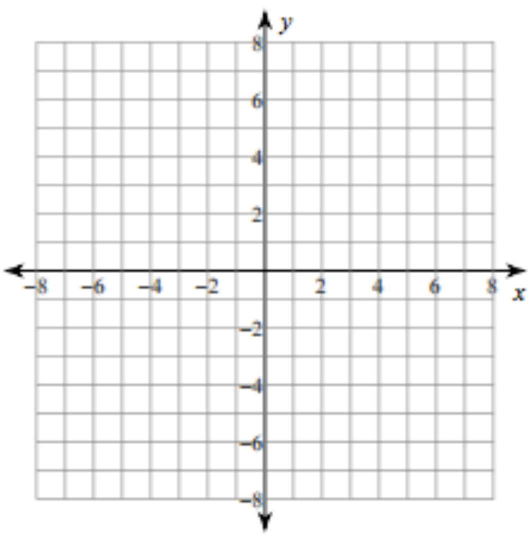
9) $f(x) = \dfrac{3x^2 - 12x}{x^2 - 2x - 3}$



10) $f(x) = \dfrac{x^3 - 16x}{-4x^2 + 4x + 24}$



11) $f(x) = \dfrac{x^2 + 2x}{-4x + 8}$



12) $f(x) = \dfrac{x + 2}{2x + 6}$